with one-dimensional nonsteady wave propagation phenomena in continuum flows suggests that solutions of the one-dimensional Grad equations for small disturbances[1, 2]

$$\frac{\partial s}{\partial t} + \frac{\partial u}{\partial x} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + \frac{p_0}{\rho_0}\frac{\partial p}{\partial x} + \frac{1}{\rho_0}\frac{\partial \tau}{\partial x} = 0 \tag{2}$$

$$\frac{\partial p}{\partial t} + \frac{5}{3}\frac{\partial u}{\partial x} + \frac{2}{3p_0}\frac{\partial q}{\partial x} = 0 \tag{3}$$

$$\frac{\partial \tau}{\partial t} + \frac{8}{15}\frac{\partial q}{\partial x} + \frac{4}{3}p_0\frac{\partial u}{\partial x} = -\frac{p_0}{\mu_0}\tau \tag{4}$$

$$\frac{\partial q}{\partial t} + \frac{p_0}{\rho_0}\frac{\partial \tau}{\partial x} + \frac{5}{2}\frac{p_0{}^2}{\rho_0}\frac{\partial \theta}{\partial x} = -\frac{p_0}{\mu_0}q \tag{5}$$

$$p = s + \theta \tag{6}$$

can be obtained in the form

$$[\partial/\partial t \pm c(\partial/\partial x)][\alpha p + \beta\theta + \gamma\tau + \delta q + \epsilon u] = 0 \tag{7}$$

Here p is the perturbation pressure, s the perturbation density, θ the perturbation temperature, u the velocity, τ the normal stress, q the heat flux, c the characteristic propagation velocity, and α, β, γ, δ, ϵ are constants. A relaxation time also can be defined by $t_f \simeq \mu_0/p_0$.

When the relaxation time is much greater than the flow time, $t_f \gg t$, Eqs. (4) and (5) can be written in the approximate form

$$\frac{\partial \tau}{\partial t} + \frac{8}{15}\frac{\partial q}{\partial x} + \frac{4}{3}p_0\frac{\partial u}{\partial x} = 0 \tag{4'}$$

$$\frac{\partial q}{\partial t} + \frac{p_0}{\rho_0}\frac{\partial \tau}{\partial x} + \frac{5}{2}\frac{p_0{}^2}{\rho_0}\frac{\partial \theta}{\partial x} = 0 \tag{5'}$$

Since solutions are sought in the form (7), the equations can be rewritten as

$$\frac{\partial p}{\partial t} - \frac{\partial \theta}{\partial t} + c_1\frac{\partial}{\partial x}\frac{u}{c_1} = 0 \tag{8}$$

$$\frac{\partial}{\partial t}\left(\frac{c_1}{c_0}\right)^2\frac{u}{c_1} + c_1\frac{\partial p}{\partial x} + c_1\frac{\partial}{\partial x}\frac{\tau}{p_0} = 0 \tag{9}$$

$$\frac{\partial p}{\partial t} + c_1\frac{\partial}{\partial x}\left(\frac{5}{3}\frac{u}{c_1}\right) + c_1\frac{\partial}{\partial x}\left(\frac{2}{3}\frac{q}{p_0c_1}\right) = 0 \tag{10}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{4}\frac{\tau}{p_0}\right) + c_1\frac{\partial}{\partial x}\left(\frac{2}{5}\frac{q}{p_0c_1}\right) + c_1\frac{\partial}{\partial x}\frac{u}{c_1} = 0 \tag{11}$$

$$\frac{\partial}{\partial t}\left(\frac{c_1}{c_0}\right)^2\frac{q}{p_0c_1} + c_1\frac{\partial}{\partial x}\frac{\tau}{p_0} + c_1\frac{\partial}{\partial x}\frac{5}{2}\theta = 0 \tag{12}$$

with

$$c_0{}^2 = p_0/\rho_0$$

On multiplying Eqs. (9–12) by the constants α, β, γ, δ, respectively, the following equations must be satisfied:

$$1 + \beta = \alpha$$

$$-1 = \tfrac{5}{2}\delta$$

$$\alpha(c_1/c_0)^2 = 1 + \tfrac{5}{3}\beta + \gamma$$

$$\tfrac{3}{4}\gamma = \alpha + \delta$$

$$\delta(c_1/c_0)^2 = \tfrac{2}{3}\beta + \tfrac{2}{5}\gamma$$

Two solutions result with propagation velocities identical with the values obtained by Grad[1] and Ai,[2] $c_1/c_0 = (0.661)^{1/2}$ and $c_1/c_0 = (4.54)^{1/2}$.

The resulting characteristic equations are

$$[(\partial/\partial t) \pm 0.813c_0(\partial/\partial x)]P_{1\pm} = 0 \tag{13}$$

$$[(\partial/\partial t) \pm 2.13c_0(\partial/\partial x)]P_{2\pm} = 0 \tag{14}$$

where

$$P_{1\pm} = [\theta - 0.51p - 0.11(\tau/p_0)] \pm (1/c_0)[0.33(q/p_0) - 0.42u]$$

$$P_{2\pm} = [\theta + 0.78p + 1.18(\tau/p_0)] \pm (1/c_0)[0.85(q/p_0) + 1.66u]$$

Assuming the existence of external heat addition $H(x, t)$ and external forces $F(x, t)^2$ and including changes in the characteristic quantities as $t \rightarrow t_f$, the equations can be written in a nondimensional form

$$\left(\frac{\partial}{\partial t} \pm 0.813\frac{\partial}{\partial x}\right)P_{1\pm} = \left(0.487\frac{H}{p_0} \mp 0.417\frac{F}{c_0}\right)\frac{L}{c_0} + \frac{L}{t_fc_0}\left(0.15\frac{\tau}{p_0} \mp 0.4\frac{q}{p_0c_0}\right) \tag{13'}$$

$$\left(\frac{\partial}{\partial t} \pm 2.13\frac{\partial}{\partial x}\right)P_{2\pm} = \left(1.78\frac{H}{p_0} \pm 1.66\frac{F}{c_0}\right)\frac{L}{c_0} - \frac{L}{t_fc_0}\left(\frac{1.57\tau}{p_0} \pm 0.4\frac{q}{p_0c_0}\right) \tag{14'}$$

on multiplying both sides of the equation by L/c_0. In Eqs. (13′) and (14′), $(c_0/x')(L/c_0)$ is replaced by $1/x$, and $(1/t') \times (L/c_0)$ is replaced by $1/t$. The validity of the characteristic approximation now can be determined from the condition $L/t_fc_0 \ll 1$, where L can be related to the distance over which the propagation occurs. The terms multiplying L/t_fc_0 provide a correction when the flow time L/c_0 approaches the relaxation time.

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Isothermal Compressibilities of Two Liquid Monopropellants

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Liquid monopropellants find wide use as gas generants for powering propellant pump turbines and as auxiliary power supplies in ballistic missiles. When the adiabatic

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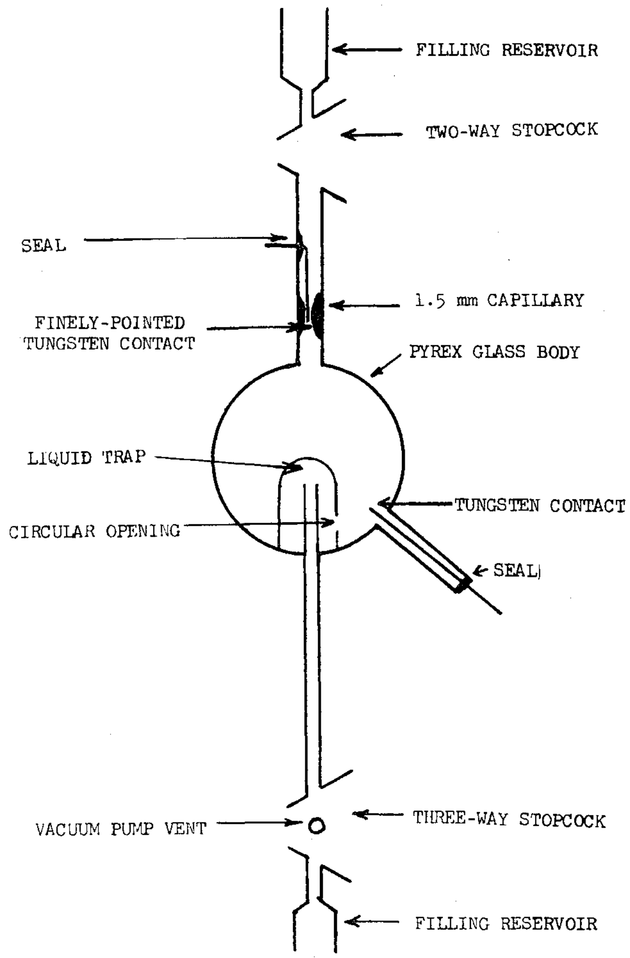

Fig. 1 Cross section of piezometer

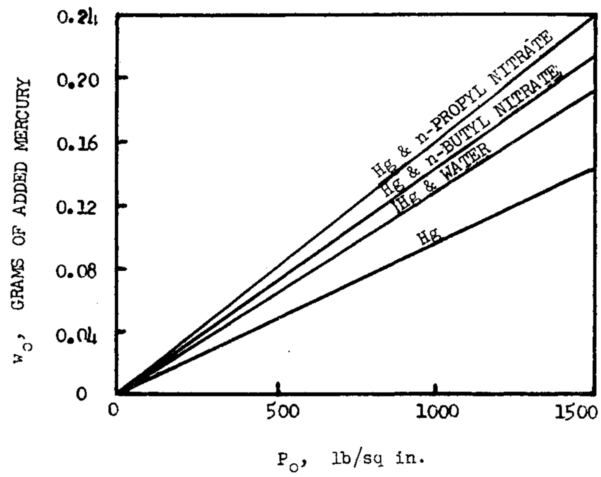

Fig. 2 Liquid isothermal compressibility data

compressibility and heat capacity of a liquid are known, liquid thermodynamic functions such as enthalpy and entropy may be calculated. The adiabatic compressibility of a liquid is an extremely difficult parameter to determine experimentally, but it can be calculated directly when both the isothermal compressibility and the ratio of specific heats for the liquid are known. Accordingly, the isothermal compressibilities of two alkyl nitrates, n-propyl and n-butyl, were determined over the relatively low pressure range of 0 to 100 atm at 25°C.

In a similar series of experiments, Kretschmar[1] measured the isothermal compressibilities of several bipropellants as an aid to studying unsteady burning phenomena resulting from transient pressure changes in injector systems.

Compressibility Method

The method of Richards and Stull[2] was chosen as the most convenient and safe method available, since the small quantities of liquid required (i.e., less than 1 cm^3) lessen the hazard of ignition during compression. By this method, the difference in compressibilities of liquid mercury and the liquid under investigation is determined from two separate series of compressions: one series on liquid mercury alone, and a second series on a mixture of liquid mercury and the liquid in question. Volume changes on compression are determined by filling a glass vessel (or piezometer) to an indicated height and recording the pressure required to depress the liquid mercury level to this height after weighed globules of mercury have been added to the vessel.

Apparatus

Figure 1 is a sketch of the glass piezometer employed in this investigation. The departure of the mercury level from the finely pointed tungsten contact was indicated by an infinite resistance reading across the two tungsten contacts. The purpose of the liquid trap was to prevent the nonconducting organic liquid from rising into the capillary section, thus indicating an open circuit between the contacts. A pressure of 40 μ Hg was maintained through the three-way stopcock during the filling procedure to insure that no air bubbles were trapped within the liquid. Before compression, the channels in the three-way stopcock were open to the exterior of the piezometer and closed to the interior. During compression, the entire piezometer was centered within a steel vessel so that it was subjected to pressure both internally and externally. The steel vessel rested in a constant-temperature water bath at 25°C.

Calculated results

Data consist of linear plots of total weight of added mercury (w_0) vs pressure (P_0) required to depress the mercury level to the finely pointed tungsten contact (see Fig. 2). One plot is for the case where the piezometer contained only mercury. Three other plots are for the case where the piezometer contained mixtures of mercury with either n-propyl nitrate, n-butyl nitrate, or distilled water. The compression on distilled water was a control run.

The form of the equation for the difference in compressibilities of mercury and the liquid in question is

$$\beta_{\text{liq}} - \beta_{\text{Hg}} = b(1 - \beta_{\text{Hg}}P_1)/(\rho W/D)$$

where β_{liq} and β_{Hg} are the compressibilities of the liquid and of mercury, respectively, between zero and some final pressure P_1, and where

b = difference in slopes from the w_0-P_0 plots for mercury and the liquid in question
ρ = density of mercury at the initial pressure
W = weight of the liquid in question in the piezometer
D = density of the liquid in question

This equation holds only in the lower pressure range, where w_0 is linear with P_0, and where the initial pressure is zero.

Table 1 Properties of alkyl nitrates at 25°C

	n-propyl nitrate	n-butyl nitrate
Molecular weight	105.09	119.12
Formula	$C_3H_7NO_3$	$C_4H_9NO_3$
Density, g/cm^3	1.0485	1.0173
Compressibility, 0 to 1500 psia, tam^{-1}	102.4×10^{-6}	68.1×10^{-6}

The compressibility of water between 0 and 100 atm is determined by this equation as follows: the difference in slopes from the mercury and mercury-water curves, b, is $(12.490 - 9.662) \times 10^{-5}$, or 2.828×10^{-5}. Also, $\beta_{Hg} = 4.009 \times 10^{-6}$ atm^{-1}, $P_1 = 1469.6$ psia, $W = 0.7303$ g, $D = 0.9970$ g/cm^3, and $\rho = 13.5340$ g/cm^3. On substituting these values in the compressibility equation, one obtains $\beta_{water} = 45.91 \times 10^{-6}$ atm^{-1}. This compares favorably with Kretschmar's value of 46.13×10^{-6} atm^{-1}. The values for the compressibilities of pure n-propyl nitrate and n-butyl nitrate at 25°C along with other pertinent properties are tabulated in Table 1.

References

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Compressibility Effects of Slender Bodies Entering Vertically into Water

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THIS note discusses vertical entry of slender bodies into water at very high speeds, in which the effect of compressibility of the water may not be neglected. Recently the incompressible counterpart of this problem was treated by Moran.[1]

If the entering body is slender, the gasdynamic equations may be linearized, resulting in the wave equation for the velocity potential ϕ in the water[2]

$$(1/c^2)\phi_{tt} - \phi_{xx} = \phi_{rr} + (1/r)\phi_r \tag{1}$$

where c is the speed of sound in the water, t the time, and (x,r) axisymmetric coordinates fixed with the free-water surface and the positive x-axis directed upward. Furthermore, by linearizing the free-surface boundary condition[3] and neglecting the effect of the Froude number (since the speed of entry U is large), the boundary condition on the free surface takes the form[1]

$$\phi = 0 \quad \text{on} \quad x = 0 \tag{2}$$

A body of revolution moving along the x-axis can be represented by a distribution of sources that vary in time along the axis of flight. The potential of these sources is represented by an integral (expressing the superposition of spherical waves emanating from each point on the axis) of the retarded values of the source strength $S(x,t)$[4]

$$-4\pi\phi(x,r,t) = \int_{-\infty}^{+\infty} \frac{S\{\xi, t - (1/c)[(x-\xi)^2 + r^2]^{1/2}}{[(x-\xi)^2 + r^2]^{1/2}} d\xi \tag{3}$$

The source strength is zero outside the body so that the integral Eq. (3) covers only those values of ξ which are common to the flight path and the surface of the retrograde Mach cone $\tau = t - [(x-\xi)^2 + r^2]^{1/2}(1/c)$. The flight-path diagram (the x,t-diagram), because of Eq. (2), will be represented as in Fig. 1. To satisfy Eq. (2), one uses the principle of source images and obtains the following condition expressing antisymmetry in x: $S(x,t) = -S(-x,t)$. The traces of the body nose and its x,t-diagram image in the will be the lines $x = -Ut$ and $x = Ut$.

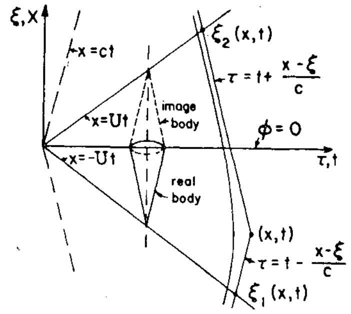

Fig. 1 Flight-path diagram for the entry problem

There will be interest in the pressure on the body surface ($x < 0$), and since the body is slender the potential expression in Eq. (3) may be expanded for small r-values. There will then be an integral for the recent past ξ_1 to ξ_2 (Fig. 1) where $\xi_1(x,t)$ and $\xi_2(x,t)$ are the intersections of the retrograde Mach cone (r small $\rightarrow$ 0) with the symmetric flight-path curves. The procedure of expanding Eq. (3) for small r is given in Ref. 4, where the resulting asymptotic expansion of the source distribution near the axis is also given. The source strength $S(x,t)$ is determined by the requirement that the flow be tangential at the surface. Within linearized theory, one obtains[4] $S(x,t) = \partial A(x,t)/\partial t$, where $A(x,t))$ is the cross-sectional area distribution of the body as a function of x and t.

The theory just outlined will be used to calculate the pressure distribution on a cone entering the water with a constant speed $U < c$. The function $S(x,t)$ becomes, then,

$$S(x,t) = 2\pi Uk^2(x + Ut), \tag{4a}$$

$$S(x,t) = -2\pi Uk^2(Ut - x), \tag{4b}$$

or

$$S(x,t) = 2\pi Uk^2\{(x + Ut)H(x + Ut)[1 - H(x)] - (Ut - x)H(x)[1 - H(x - Ut)]\} \tag{5}$$

where $k = \tan\theta_0$, θ_0 is the semiangle at the vertex of the cone, and $H(x)$ is the Heaviside unit step function. The two points ξ_1 and ξ_2 are given by $\xi_1 = -(Ut - Mx)/(1 + M)$, $\xi_2 = (Ut + Mx)/(1 + M)$ where $M = U/c$. Defining a pressure coefficient c_p as $(p - p_s)/(\rho U^2/2)$, where p_s is the pressure at the free surface, and assuming for the water that $p\rho^{-n}$ = const, one obtains the usual linearized pressure formula

$$c_p = -(2/U^2)\phi_t - (1/U^2)\phi_r^2 + \ldots \tag{6}$$

By calculating c_p from Eq. (6), one obtains for $-Ut < x < 0$

$$c_p = k^2\left[\log\frac{4x^2}{k^2(U^2t^2 - x^2)} - 1\right] \tag{7}$$

As is seen from Eq. (7), there is no Mach-number effect on the c_p-distribution for the cone.

Next, a drag coefficient c_F is defined by (F = force on the body)

$$c_F = \frac{F}{1/2\rho U^2\pi(Utk)^2} = \frac{2}{(Ut)^2}\int_{-Ut}^{0} c_p(Ut + x)dx \tag{8}$$

For the cone, one gets

$$c_F = 2k^2[\log(1/2k) - (1/2)] \tag{9}$$

Because of the suction region at the shoulder, the theory breaks down completely for cone angles $\theta_0 > 16.9°$, where c_F becomes negative.

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